Newton gave a power series solution for the initial-value problem

$$\frac{dy}{dx} = 1 - 3x + y + x^2 + xy, \qquad y(0) = 0. \tag{1}$$

Notice that the differential equation is linear and first order. By the existence and uniqueness theorem for linear equations, this problem has a solution defined on the interval $(-\infty, \infty)$. Putting the differential equation into standard form

$$\frac{dy}{dx} - (1+x)y = 1 - 3x + x^2 \tag{2}$$

we notice that

$$\mu(x) = e^{-1/2}e^{-x-x^2/2} = e^{-(x+1)^2/2}$$

is an integrating factor. That is, after multiplying both sides of (2) by $\mu(x)$ we have

$$\frac{d}{dx}(\mu(x)y(x)) = \mu(x)(1 - 3x + x^2).$$

Thus μy is an antiderivative of

$$\begin{aligned}\mu(x)(1 - 3x + x^2) &= \mu(x)((x+1)^2 - 5x)\\ &= \mu(x)((x+1)^2 - 5(x+1) + 5).\end{aligned}$$

Under the substitution $t = x + 1$ we have

$$\int \mu(x)(1 - 3x + x^2)\,dx = \int e^{-t^2/2}(t^2 - 5t + 5)\,dt. \tag{3}$$

Expanding the right-hand side and integrating the first term by parts gives

$$\begin{aligned}\int t^2 e^{-t^2/2}\,dt &= \int (t)(te^{-t^2/2})\,dt\\ &= -te^{-t^2/2} + \int e^{-t^2/2}\,dt\end{aligned}$$

and thus the right-hand side of (3) reduces to

$$-te^{-t^2/2} + \int e^{-t^2/2}\,dt - 5\int te^{-t^2/2}\,dt + 5\int e^{-t^2/2}\,dt = (5-t)e^{-t^2/2} + 6\int e^{-t^2/2}\,dt.$$

Recall that the error function is defined by

$$\operatorname{erf}(\beta) = \frac{2}{\sqrt{\pi}}\int_0^\beta e^{-\xi^2/2}\,dt.$$

A straightforward substitution gives the general antiderivative

$$\int e^{-t^2/2}\,dt = \sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(t/\sqrt{2}\right) + C.$$

Thus there exists a constant C such that

$$\begin{aligned}\mu(x)y(x) &= e^{t^2/2}y(x)\\ &= (5-t)e^{-t^2/2} + 6\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(t/\sqrt{2}\right) + C\end{aligned}$$

and hence

$$\begin{aligned}y(x) &= 5 - t + e^{t^2/2}\left(3\sqrt{2\pi}\,\mathrm{erf}\left(t/\sqrt{2}\right) + C\right)\\ &= 4 - x + e^{(x+1)^2/2}\left(3\sqrt{2\pi}\,\mathrm{erf}\left((x+1)/\sqrt{2}\right) + C\right).\end{aligned} \tag{4}$$

The initial condition from (1) forces

$$0 = 4 + e^{1/2}\left(3\sqrt{2\pi}\,\mathrm{erf}\left(1/\sqrt{2}\right) + C\right)$$

which implies

$$C = -4e^{-1/2} - 3\sqrt{2\pi}\,\mathrm{erf}\left(1/\sqrt{2}\right). \tag{5}$$

Thus from (4) and (5), the solution of (1) is given by

$$y(x) = 4 - x + e^{(x+1)^2/2}\left\{3\sqrt{2\pi}\left[\mathrm{erf}\left((x+1)/\sqrt{2}\right) - \mathrm{erf}\left(1/\sqrt{2}\right)\right] - 4e^{-1/2}\right\}.$$